Asset and Risk Management I

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Parametric Portfolio Management

Variation in Expected Returns

Expected returns and risk premia for various asset classes and for individual securities are **not constant over time**.

- Portfolio optimization should take this into account!
- **Traditional way**: Estimate the time-varying expected risk premia and then optimize, e.g., using a Markowitz model.
- **Problem**: Time-varying expected risk premia are hard to estimate and the optimal portfolio weights respond very sensitively to changes in these expectations (i.e., portfolio weights change dramatically even for only small changes in expected risk premia).

Solutions to Variation in Expected Returns

- Black and Litterman (1992) approach:
  - Take estimation error in expected risk premia explicitly into account when constructing your portfolio.
- Brandt, Santa-Clara, and Valkanov (2009) approach:
  - Skip estimating expected risk premia altogether.
  - Instead, optimize the portfolio directly as a function of the predictive variables (e.g., momentum, book-to-market, etc.).

Main Idea of BSV

The main idea of parametric portfolio optimization (by BSV) is:

- Specify the portfolio weight function as:

$$w_{i,t} = f(x_{i,t}; \theta),$$

  where

  - $f(.;.)$ is a known function,
  - $x_{i,t}$ is a vector of conditioning information (possibly firm-specific), and
  - θ are parameters to be estimated.

Parameterized Portfolio Weights

- The portfolio weight function is

$$\begin{aligned} w_{i,t} &= f(x_{i,t};\theta) \\ &= \bar{w}_{i,t} + \frac{\theta' x_{i,t}}{N_t}, \end{aligned}$$

  where

- $\bar{w}_{i,t}$ weight of stock i in the benchmark (or market) portfolio at time t,
- N_t number of stocks at time t, and
- θ vector of coefficients to be estimated.

Characteristics

- Vector of characteristics for each firm $x_{i,t}$.
- The characteristics are normalized across **stocks** (mean zero and standard deviation one) at **each time** t.
- An example for the potential characteristics:
  - Market capitalization (*me*),
  - Book-to-market ratio (*btm*), and
  - Lagged twelve-month return (*mom*).
- Then, the portfolio weight function is

$$w_{i,t} = \bar{w}_{i,t} + \frac{\theta_{me} \cdot me_{it} + \theta_{btm} \cdot btm_{it} + \theta_{mom} \cdot mom_{it}}{N_t}.$$

Portfolio Returns

- Given the portfolio weight function, we compute portfolio returns

$$
\begin{aligned}
r_{p,t+1} &= \sum_{i=1}^{N_t} w_{i,t} r_{i,t+1} \\
&= \sum_{i=1}^{N_t} \left(\bar{w}_{i,t} + \frac{\theta' x_{i,t}}{N_t} \right) r_{i,t+1} \\
&= r_{m,t+1} + r_{a,t+1},
\end{aligned}
$$

where

- $w_{i,t}$ is a function of predetermined variables,
- $r_{m,t+1}$ is the market (or benchmark) portfolio return, and
- $r_{a,t+1}$ is the return of an actively managed (hedge) portfolio.

Comments

- Coefficients θ are constant across stocks and through time.
  - Portfolio policy only cares about characteristics, and
  - Unconditional portfolio policy instead of conditional portfolio weights.
- Standardized characteristics mean that
  - $\sum_{i=1}^{N_t} \frac{\theta' x_{i,t}}{N_t} = 0$.
  - Therefore, we have deviations from the benchmark (active portfolio management), and
  - Portfolio weights are guaranteed to sum to one.
- Normalization $\frac{1}{N_t}$ allows number of firms to change over time.

The Optimization Problem

- Investors problem: Choose weights to maximize expected utility

$$\max_{\{w_{i,t}\}_{i=1}^{N_t}} \mathbb{E}\left[u(r_{p,t+1})\right] = \mathbb{E}\left[u\left(\sum_{i=1}^{N_t} w_{i,t} r_{i,t+1}\right)\right].$$

- The expected return is assumed to depend on a vector of firm or asset-class characteristics $\boldsymbol{x_{i,t}}$
- A linear specification of the weight function

$$w_{i,t} = \bar{w}_{i,t} + \frac{1}{N_t}\boldsymbol{\theta}'\,\boldsymbol{\hat{x}_{i,t}}$$

Benchmark weights

Constant coefficient vector to be estimated

Standardized stock or asset class characteristics ($\mu = 0, \sigma = 1$)

Possible Specifications

- If stock or asset class characteristics capture all relevant aspects of the joint return distribution

$$\max_{\boldsymbol{\theta}} \frac{1}{T} \sum_{t=0}^{T-1} u\left(\sum_{i=1}^{N_t}\left(\bar{w}_{i,t} + \frac{1}{N_t}\boldsymbol{\theta}' \hat{\boldsymbol{x}}_{\boldsymbol{i,t}}\right) r_{i,t+1}\right).$$

- No short-sales possible:

$$w_{i,t}^{+} = \frac{\max\{0, w_{i,t}\}}{\sum_{j=1}^{N_t}\max\{0, w_{j,t}\}}.$$

- Constant relative risk aversion (CRRA)

$$u(r_{p,t+1}) = \frac{(1 + r_{p,t+1})^{1-\gamma}}{1-\gamma}.$$

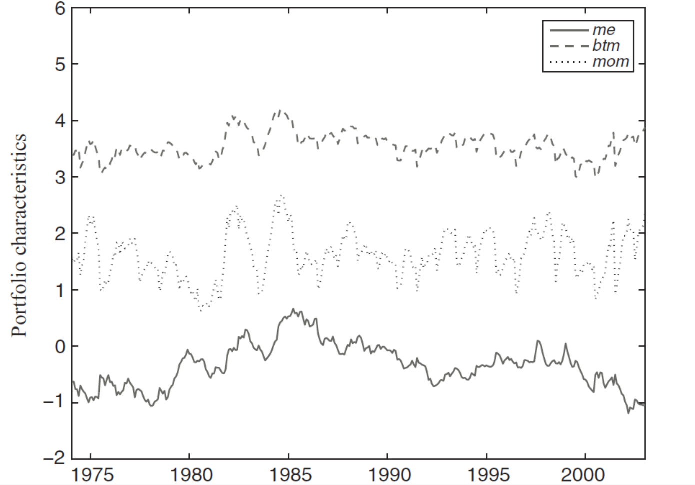
Main Results

- Compared to market capitalization optimization gives higher weights to
  - Stocks of small firms,
  - Firms with high book-to-market ratio (value firms), and
  - Firms with high lagged returns (winners).
- With $\gamma = 5$, the certainty-equivalent gain from optimization is 11.1% p.a. in sample, 5.4% out of sample.

▸ CE gain

- With high risk aversion,
  - Coefficients on size and momentum approach zero, and
  - Value coefficients might be unrelated to risk.

Portfolio Characteristics $\left(\sum_{i=1}^{N_t} w_{i,t} x_{i,t}\right)$ Over Time (CRRA, $\gamma = 5$)



Variable	VW	EW	In Sample PPP	Out of Sample PPP
θ_{me}	–	–	−1.451	−1.124
Std. err.	–	–	(0.548)	(0.709)
θ_{btm}	–	–	3.606	3.611
Std. err.	–	–	(0.921)	(1.110)
θ_{mom}	–	–	1.772	3.057
Std. err.	–	–	(0.743)	(0.914)
LRT p-value	–	–	0.000	0.005
$\lvert w_i \rvert \times 100$	0.023	0.023	0.083	0.133
$\max\ w_i \times 100$	3.678	0.023	3.485	4.391
$\min\ w_i \times 100$	0.000	0.023	−0.216	−0.386
$\sum w_i I(w_i < 0)$	0.000	0.000	−1.279	−1.447
$\sum I(w_i \leq 0)/N_t$	0.000	0.000	0.472	0.472
$\sum \lvert w_{i,t} - w_{i,t-1} \rvert$	0.097	0.142	0.990	1.341
CE	0.064	0.069	0.175	0.118
$\bar{r}$	0.139	0.180	0.262	0.262
$\sigma(r)$	0.169	0.205	0.188	0.223
SR	0.438	0.564	1.048	0.941
α	–	–	0.174	0.177
β	–	–	0.311	0.411
$\sigma(\epsilon)$	–	–	0.181	0.214
IR	–	–	0.960	0.829
me	2.118	−0.504	−0.337	−0.029
btm	−0.418	0.607	3.553	3.355
mom	0.016	0.479	1.623	2.924

Appendix Parametric Optimization

Certainty-equivalent Gain

- The certainty equivalent translates a risky portfolio return into a risk-adjusted return for an investor with risk aversion γ.
  - For utility function $u(\cdot)$ and initial wealth W_0, the certainty-equivalent return r_p^{CE} is defined implicitly by

$$u\big(W_0(1 + r_p^{CE})\big) = \mathbb{E}[u(W_0(1 + r_{p,t}))] \, .$$

  - With mean–variance preferences:

$$r_p^{CE} = \mathbb{E}[r_{p,t}] - \frac{\gamma}{2} \mathrm{Var}(r_{p,t}).$$

- The certainty-equivalent gain ΔCE from optimization is:

$$\Delta CE = r_{opt}^{CE} - r_{bm}^{CE}.$$

  - Interpretation: The investor would be willing to give up ΔCE of certain return to switch from portfolio bm to opt.

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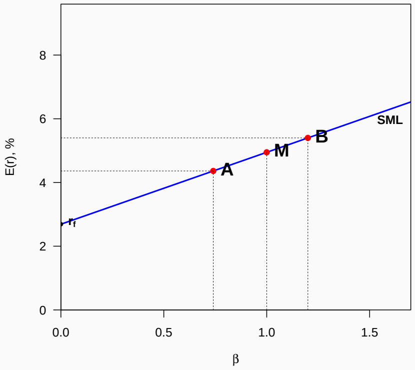
Performance Measurement

Overview

- Measuring performance with asset pricing models:
  - One factor: Jensen's alpha, Treynor index, Sharpe ratio, M^2, and upmarket-downmarket beta
  - Multiple factors: APT alpha and style analysis
- Measuring performance without asset pricing models: Grinblatt-Titman measure, tracking error, and information ratio
- Attribution analysis

CAPM-World

Benchmark in the CAPM-World



- If the market is in equilibrium and no manager has superior information, then all the funds must be on one line (SML).
- The expected return should only depend on the beta factor.
- Therefore, the appropriate benchmark is: $r_{bmk} = r_f + \beta_P(\mathbb{E}[r_M - r_f])$

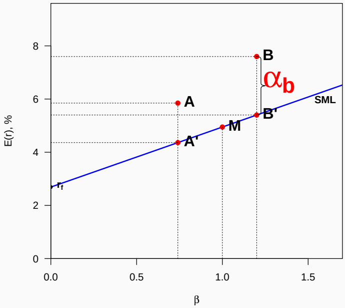
Jensen's Alpha (1)

Example: Fund *A* has a beta of 0.74

- Return data in %:

Quarter	T-Bill	Fund A		S&P 500	
	r_t	r_t	r_t^e	r_t	r_t^e
Q1	2.97	-2.77	-5.74	-5.86	-8.83
Q2	3.06	1.03	-2.03	-2.94	-6.00
Q3	2.85	11.14	8.29	13.77	10.92
Q4	1.88	13.99	12.11	14.82	12.94
Mean	2.69	5.85	3.16	4.95	2.26
Annual Mean	10.76	23.39	12.63	19.79	9.03
SD	0.55	8.00	8.42	10.87	11.26
Annual SD	1.09	15.99	16.84	21.74	22.52

Jensen's Alpha



- If the market is in equilibrium and no manager has superior information, all funds must be on one line (SML).
- The expected return should only depend on the beta factor.
- If a fund manager has superior performance, then Fund P lies above the SML.
- The distance to the SML is the alpha:

$$\alpha_P = \mathbb{E}[r_P - r_f] - \beta_P(\mathbb{E}[r_M - r_f])$$

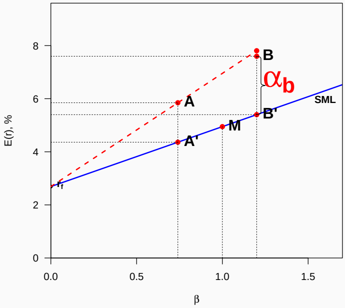
Example

Example cont.: Questions

- What was the alpha for Fund A?
- Suppose fund B has a beta of 1.2 and an alpha of 2.2%. How would you rank fund A and fund B?
- By how much could one lever fund A to achieve a beta of 1.2?
- What is Jensen's Alpha for the levered fund?
- How would we now rank Fund A versus Fund B?

Solution

Treynor Index



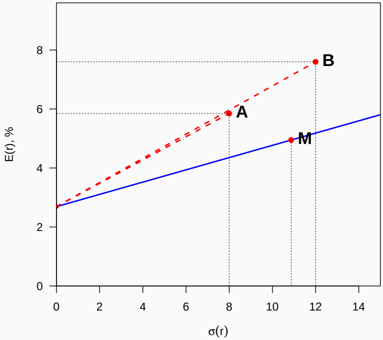
- Measures the excess return per unit of systematic risk (i.e., beta).
- Considers the leverage effect

$$\text{Treynor} = \frac{\mathbb{E}[r_P - r_f]}{\beta_P}$$

Treynor Index (2)

Example: Treynor

- Treynor Index for fund A: $\frac{3.16\%}{0.74} = 4.26\%$
- Treynor Index for fund B: $\frac{4.91\%}{1.2} = 4.09\%$
  Note that the excess rate of return of fund B must have been $2.2\% + 2.26\% \times 1.2 = 4.91\%$
- Treynor Index for fund $S\&P$ 500: 2.26%
  (=average excess return of the market)



- Divides the excess return by the volatility.
- σ_P is frequently defined as the volatility of the excess rate of return.
- Graphically, it is the slope of the lines from r_f to A or B.
- Ratio of return to risk

$$\text{Sharpe} = \frac{\mathbb{E}[r_P - r_f]}{\sigma_P}$$

Example: Sharpe Ratio

Using data from the previous example, calculate the Sharpe Ratio for Fund A and the S&P 500.

- Sharpe ratio for Fund A
- Sharpe ratio for the S&P 500
- Was there superior performance?

Solution

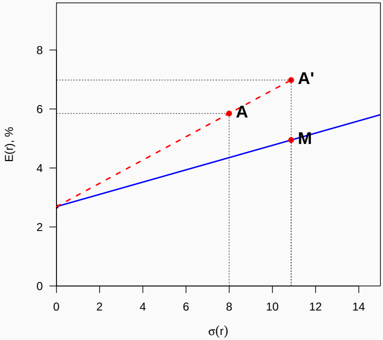
Sharpe Ratio: Additional Considerations

- The Sharpe ratio should be expressed on an annual basis:

$$\text{Sharpe Ratio Annual} = \frac{\mathbb{E}[r_P - r_f] \times 4}{\sigma_P \sqrt{4}}$$

- The higher the Sharpe ratio, the better the portfolio performance because either
  - the return was higher or
  - the risk was smaller.
- Comparison with the Sharpe ratio of the market.
- Note: The Sharpe ratio ignores correlations between the fund and clients' other investments.

Modigliani-Modigliani (M^2) Measure



- Easier to interpret than Sharpe Ratio.
- Return of a fund with the same risk as the benchmark.
- Includes leverage effect

$$M_P^2 = r_f + \frac{\mathbb{E}[r_P - r_f]}{\sigma_P}\sigma_M$$

Modigliani-Modigliani (M^2) Measure (2)

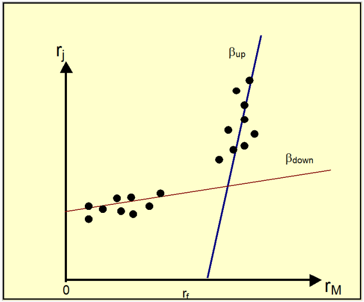
Example: M^2

- Calculate M^2 for Fund A.

- According to M^2, did Fund A have superior performance?

Solution

Upmarket and Downmarket Betas



- Estimating two regression coefficients β_{up} and β_{down}
- Market timing expertise exists when $\beta_{up} - \beta_{down} > 0$

Summarizing CAPM-based PMs

- Sharpe Ratio, M^2:
  - Consider return and total risk (volatility).
  - Return per unit of total risk.
  - Assumption: Entire wealth is invested in the fund.
- Alpha, Treynor:
  - Consider return and systematic risk.
  - The idiosyncratic risk of a stock is ignored.
  - Only systematic risk counts.
  - Assumption: Only a small part of the investor's wealth is invested in the fund.

APT-based Performance Measurement

APT-based PM (1)

- Stock returns are related to basic factors:

$$r_{i,t} = a_i + \beta_{i,1} F_{1,t} + \beta_{i,2} F_{2,t} + \cdots + \beta_{i,L} F_{L,t} + \epsilon_{i,t}$$

- Expected returns are linearly related to factor exposures:

$$\mathbb{E}[r_i] = r_f + \beta_{i,1}\lambda_1 + \beta_{i,2}\lambda_2 + \cdots + \beta_{i,L}\lambda_L$$

- Performance measurement using APT:

$$\text{Alpha}_{APT} = \mathbb{E}[r_P] - [r_f + \beta_{i,1}\lambda_1 + \beta_{i,2}\lambda_2 + \cdots + \beta_{i,L}\lambda_L]$$

APT-based PM (2)

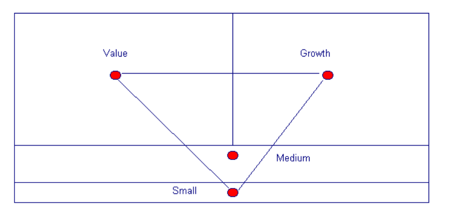
- A popular model used in practice is the Fama-French Three Factor Model:
  - Market factor,
  - Size factor, and
  - Book-to-market (value) factor.
- The momentum factor is also often considered (Four Factor Model)
- Problems with performance measurement using APT:
  - Factors are not theoretically specified, and
  - Factor structure not uniquely determined.

Style Analysis: Basics

- Asset returns follow a factor model, i.e.,

$$r_{i,t} = a_i + \beta_{i,1} F_{1,t} + \beta_{i,2} F_{2,t} + \cdots + \beta_{i,L} F_{L,t} + \epsilon_{i,t}$$

- Asset class factor model: Each factor k is the return of an asset class and $\sum_k \beta_{i,k} = 1$.
- Choice of basic asset classes:



Source: Sharpe (1992), Figure 1

Style Analysis: Estimation

- Method introduced by Sharpe (1992): Asset Allocation: Management Style and Performance Measurement, The Journal of Portfolio Management 18(2).
- Factor exposures can be estimated by
  - Regression
  - Constrained Regression
    - Sum of factor loadings $= 1$
  - Quadratic programming: minimize the variance of return difference with constraints
    - Sum of factor loadings $= 1$
    - Individual loading ≥ 0

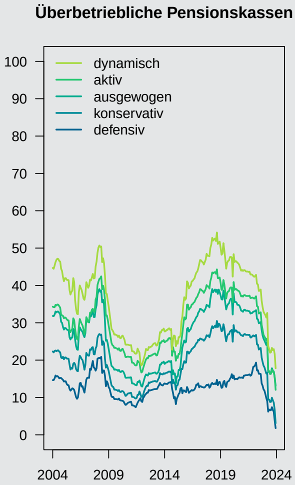
Style Analysis: Example

Example: A U.S. Mutual Fund

	Reg.	Const.	Quadr.
Bills	14.69	42.65	0.
Int. Bonds	-69.51	-68.64	0.
LT Bonds	-2.54	-2.38	0.
Corp. Bonds	16.57	15.29	0.
Mortgages	5.19	4.58	0.
Value Stocks	109.53	110.35	69.81
Growth Stocks	-7.86	-8.02	0.
Medium Stocks	-41.83	-43.62	0.
Small Stocks	45.65	47.17	30.04
Foreign Bonds	-1.85	-1.38	0.
Europ. Stocks	6.15	5.77	0.15
Japan. Stocks	-1.46	-1.79	0.
Total	72.71	100.00	100.00
R^2	95.20	95.16	92.22

- Both unconstrained and constrained regression give problematic results.
- Quadratic programming is the recommended estimation method.

Example: Style Analysis over Time



- Style analysis over rolling windows can help to understand dynamic asset allocation decisions.
- Example: Austrian pension funds' allocation to equity over time.
- This shows the economic, not accounting exposure; e.g., it takes derivatives into account.
- However, results can be poor if the asset classes are not well matched.

Source: Randl and Zechner (2025)

Alternative Measures

Grinblatt-Titman Measure

- A skilled manager will increase the weights of asset classes with increasing expected returns.
- Portfolio Change Measure

$$PCM = \sum_{t=2}^{T} \sum_{j=1}^{N} \frac{r_{j,t+1}(w_{j,t} - w_{j,t-1})}{T-1}$$

- Does not require a benchmark portfolio but observable portfolio weights.
- Portfolio holdings of an uninformed investor are uncorrelated with future asset returns.
- Positive correlation between
  - Changes in portfolio weights and
  - Return from t to $t+1$.

Tracking Error and Information Ratio

- Active portfolio management is evaluated relative to a benchmark, $r_t^a = r_{P,t} - r_{B,t}$, where r_t^a is the active return.
- Average relative performance: $\bar{r}^a = \frac{1}{T}\sum_{t=1}^{T}(r_{P,t} - r_{B,t})$
- Tracking error: $TE_P = \sigma(r_{P,t} - r_{B,t}) = \sigma(r_t^a)$
- Information ratio: $IR_P = \frac{\bar{r}^a}{TE_P}$
- Relative risk: $\frac{\sigma(r_P)}{\sigma(r_B)}$

Comparison with Model-based Measures

- If the benchmark return is $r_f + \beta_P \cdot r_M^e$,
  i.e., the benchmark is a portfolio of the risk-free asset and the market with the same beta as the portfolio, then
  - the mean Relative Performance equals Jensen's Alpha,
  - the Tracking Error is the non-systematic risk of the portfolio, $\sigma(\epsilon_p)$, and
  - the Information Ratio equals $\frac{\alpha_P}{\sigma(\epsilon_p)}$.

Interpreting the Relative Performance

- Risk relative to benchmark $= \frac{\sigma(\text{Portfolio})}{\sigma(\text{Benchmark})}$
- $R^2 =$ Squared correlation coefficient between the rate of return on the portfolio and the rate of return on the benchmark. Represents the fraction of portfolio risk that can be explained by the benchmark.

The Inference Problem

- How many months of data do we need if we want to be quite sure that outperformance is not just luck?
- Regression-based estimation of Jensen's Alpha gives both
  - a point estimate $\hat{\alpha}_P$ and
  - its standard error $\hat{\sigma}(\hat{\alpha}) \approx \frac{\hat{\sigma}(\epsilon)}{\sqrt{N}}$
- The t-statistic is $t(\hat{\alpha}) = \frac{\hat{\alpha}}{\hat{\sigma}(\alpha)} = \frac{\hat{\alpha}\sqrt{N}}{\hat{\sigma}(\epsilon)}$

Example: Skill or Luck?

If we estimate for fund F a monthly Alpha of 0.2%, residual risk of 2%, and require 5% significance, we have to solve $1.96 = \frac{0.2\sqrt{N}}{2}$ (for large N). We need ≈ 384 months of data to be confident the alpha is not just luck.

Summary

- Returns must be adjusted by risk!
- Asset pricing models give some guidance on how to adjust for risks.
- However, we do not know exactly what is the right asset pricing model and how to implement it.
- Measuring performance without asset pricing models is possible if portfolio holding data are available or if a benchmark is pre-specified.
- Style analysis is a tool to back out a suitable benchmark from the data.

Attribution Analysis

What is Attribution Analysis?

- Relative performance is the difference between the portfolio return and the benchmark return.
- Attribution analysis studies the source of relative performance, i.e., the deviation of the portfolio return from the benchmark return.

Performance Attribution: The Base Case

	Portfolio returns R_j^p	Benchmark returns R_j^b
Portfolio weights w_j^p	$R_P = \sum_j w_j^p R_j^p$	$R_{AF} = \sum_j w_j^p R_j^b$
Benchmark weights w_j^b	$R_{SF} = \sum_j w_j^b R_j^p$	$R_B = \sum_j w_j^b R_j^b$

For comparison, we calculate returns of an Active Allocation Fund AF and Active Security Selection Fund SF. The relative performance of portfolio P to benchmark B can be written as:

$$R_P - R_B = \underbrace{(R_{AF} - R_B)}_{\text{Asset allocation}} + \underbrace{(R_{SF} - R_B)}_{\text{Security selection}} + \underbrace{(R_P - R_{AF} - R_{SF} + R_B)}_{\text{Interaction}}$$

$$= \sum_j (w_j^p - w_j^b) R_j^b + \sum_j w_j^b (R_j^p - R_j^b) + \sum_j (w_j^p - w_j^b)(R_j^p - R_j^b).$$

Asset Allocation and Security Selection

- Asset allocation $= R_{AF} - R_B$:
  contribution to performance due to over- or under-weighting sectors relative to the benchmark.
- Security selection $= R_{SF} - R_B$:
  contribution to performance due to earning portfolio sector returns that differ from benchmark sector returns.
- Interaction $= R_P - R_{AF} - R_{SF} + R_B$:
  residual effect from simultaneously changing sector weights and sector returns.
- Total value added $= R_P - R_B$:
  the relative performance, sum of asset allocation, security selection, and interaction.

Performance Attribution: Example

Example: Performance Attribution

Sector	Benchmark		Portfolio	
	Weight	Return	Weight	Return
Utilities	20%	2.00%	10%	2.00%
Financials	30%	3.00%	30%	4.00%
Tech	50%	4.00%	60%	9.00%

- What is the relative performance?
- What are the contributions of asset allocation and security selection?
- How big is the interaction term?

Solution

Solutions

Example

Solution (1)

- What was the alpha for Fund A?

  $\bar{r}^e_{Fund} - \beta \bar{r}^e_{S\&P} = 3.16\% - 0.74 \cdot 2.26\% = 1.49\%$

- Suppose fund B has a beta of 1.2 and an alpha of 2.2%. How would you rank fund A and fund B?

  $2.2\% > 1.49\% \Rightarrow B$ is better than A.

- By how much could one lever fund A to achieve a beta of 1.2?

  Invest 162% in A, financed by 62% shorting the risk free asset.

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Example cont.

Solution (2)

- What is Jensen's Alpha for the levered fund?

$$\begin{aligned}\bar{r}^e_{Fund} - \beta \bar{r}^e_{S\&P} &= 1.62 \cdot 3.16\% - (1.62 \cdot 0.74) \cdot 2.26\% \\ &= 1.62 \cdot 1.49\% = 2.41\%\end{aligned}$$

- How would we now rank Fund A versus Fund B?

  $2.2\% < 2.41\% \Rightarrow B$ is *worse* than the levered position in A.

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Sharpe Ratio (3)

Example: Sharpe Ratio

Using data from the previous example, calculate the Sharpe Ratio for Fund A and the S&P 500.

- Sharpe ratio for Fund A

  $\bar{r}^e_{Fund}/\sigma_{r^e_{Fund}} = 3.16/8.42 = 0.38$

  or: $\bar{r}^e_{Fund}/\sigma_{r_{Fund}} = 3.16/8 = 0.39$
- Sharpe ratio for the S&P 500

  with vol of excess returns: $2.26/11.26 = 0.2$

  with vol of returns: $2.26/10.87 = 0.21$
- Was there superior performance?

  Yes. But keep in mind that typically there is estimation error.

Back

Modigliani-Modigliani (M^2) Measure (2)

Example: M^2

- Calculate M^2 for Fund A.

  $M^2_{Fund} = 2.69\% + 3.16\% \cdot \frac{11.26\%}{8.42\%} = 6.91\%$

- According to M^2, did Fund A have superior performance?

  Compare with index performance:

  $M^2_{Fund} = 6.91\% > 4.95\% = r_{SPX}$

  Yes, there is superior performance.

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Solution: Performance Attribution

Sector	w_j^b	R_j^b	w_j^p	R_j^p	R_B	R_P	R_{AF}	R_{SF}
Utilities	20%	2.00%	10%	2.00%	0.40%	0.20%	0.20%	0.40%
Financials	30%	3.00%	30%	4.00%	0.90%	1.20%	0.90%	1.20%
Tech	50%	4.00%	60%	9.00%	2.00%	5.40%	2.40%	4.50%
Total	100%		100%		3.30%	6.80%	3.50%	6.10%

- Relative performance: $R_P - R_B = 6.80\% - 3.30\% = 3.50\%$.
- Attribution:

$$\begin{aligned}
\text{Asset allocation} &= R_{AF} - R_B = 3.50\% - 3.30\% = 0.20\%, \\
\text{Security selection} &= R_{SF} - R_B = 6.10\% - 3.30\% = 2.80\%, \\
\text{Interaction} &= R_P - R_{AF} - R_{SF} + R_B \\
&= 6.80\% - 3.50\% - 6.10\% + 3.30\% = 0.50\%.
\end{aligned}$$

- Sum: $0.20\% + 2.80\% + 0.50\% = 3.50\%$.

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